Statistics

Lecture 12

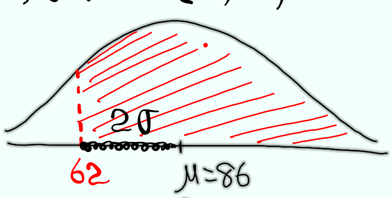
Feb 19-8:47 AM

Class Quiz 7

Drawing, labeling, shading, and full TI Command required.

Consider a normal prob. dist. with the mean of 86 and standard dev. of 12. $N(86, 12)$

1) Find $P(x > 62)$



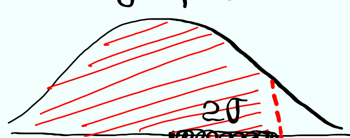
$\mu = 86$

$\sigma = 12$

$= \text{normalcdf}(62, E99, 86, 12)$

$= \boxed{.977}$

2) Find $P(\bar{x} < 98)$ for group of 4.



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 86 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{4}} = 6 \end{cases}$

$= \text{normalcdf}(-E99, 98, 86, 6)$

$= \boxed{.977}$

Nov 15-7:20 AM

SG 22 & SG 23

Estimating Parameters

Sample ← Statistic

Population ← Parameter

To estimate parameters, we must use Similar Statistic.

To estimate	we must use
Pop. Proportion p	Sample Proportion $\hat{p}$ (P-hat)
Pop. Mean μ	Sample Mean $\bar{x}$

Point-estimate

$\hat{p}$ is point-estimate for p

$\bar{x}$ is " " " μ

Nov 15-11:48 AM

When estimating Parameter, the answer would be a $\underbrace{\text{range of values}}_{\text{Confidence Interval}}$.

Every Conf. interval comes with $\underbrace{\text{Confidence level}}$. (C-level)

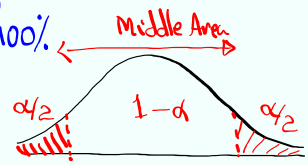
Middle Area

Middle Region

$(1-\alpha)\cdot 100\%$

Alpha

$0 < \alpha < 1$



α is Significance level

$\alpha/2$ is the area of each tail.

When C-level not given ⟹ use 95%

When α not given ⟹ use .05

Nov 15-11:55 AM

Confidence Interval for Population Proportion P:

$$\hat{p} - E < P < \hat{p} + E$$

Point-estimate

Sample Proportion

Margin of error

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\hat{p} = \frac{x}{n}$$ ← # of favorable responses ← Sample Size

$$\hat{q} = 1 - \hat{p}$$

Critical Value for $(1-\alpha)\cdot 100\%$ C-level.

Nov 15-12:02 PM

I surveyed 100 students and 80 of them voted in presidential election. Find $\boxed{99\%\text{ Conf. interval}}$ for the Prop. of all students that voted in last election.

C-level: .99

$n = 100$

$x = 80$

$\hat{p} = \frac{x}{n} = \frac{80}{100} = .8$

$\hat{q} = 1 - \hat{p} = .2$

$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

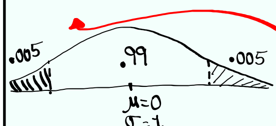
$= 2.576 \sqrt{\frac{(.8)(.2)}{100}} \approx \boxed{.1}$

$\hat{p} - E < P < \hat{p} + E$

$.8 - .1 < P < .8 + .1$

$\boxed{.7 < P < .9}$

We are 99% confident that between 70% & 90% of all students voted in the last election.



$Z_{\alpha/2} = \text{invNorm}(.995, 0, 1) = \boxed{2.576}$

Using TI:

$\boxed{\text{STAT}}$ $\boxed{\text{TESTS}}$ $\boxed{\text{1-PropZInt}}$

x: 80

n: 100

C-level: .99

$\boxed{\text{Calculate}}$

$.7 < P < .9$

$E = \frac{.9 - .7}{2} = .1$

$\hat{p} = \frac{.9 + .7}{2} = .8$

Nov 15-12:07 PM

I Surveyed 250 Students and 40 were Smokers.

C-level: .9

Find $\boxed{90\%\text{ Conf. interval}}$ for the prop. of all Students that are Smokers.

$n = 250$

$x = 40$

$\hat{p} = \frac{x}{n} = \frac{40}{250} = .16$

$\hat{q} = 1 - \hat{p} = .84$

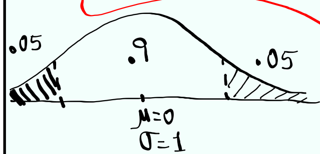
$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 1.645 \cdot \sqrt{\frac{(.16)(.84)}{250}} \approx \boxed{.04}$

$\hat{p} - E < p < \hat{p} + E$

$.16 - .04 < p < .16 + .04$

$\boxed{.12 < p < .20}$



$Z_{\alpha/2} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

1 - PropZInt

$x = 40$

$n = 250$

C-level: .9

Calculate

$E = \frac{.2 - .12}{2} = \boxed{.04}$ $\quad \hat{p} = \frac{.2 + .12}{2} = \boxed{.16}$

Nov 15-12:21 PM

I Surveyed 240 Registered Voters and 72% of them trusted the outcom of election.

$n = 240$

$\hat{p} = .72$

$\hat{p} = \frac{x}{n}$

$x = n\hat{p} = 240(.72) = 172.8 \rightarrow \boxed{x = 173}$

if decimal → Round up

Find $\boxed{99\%\text{ Conf. interval}}$ for the prop. of all Voters that trust the outcom of election

C-level: .99

1 - Prop Z Int

$x = 173$

$n = 240$

C-level: .99

Calculate

$.65 < p < .80$

$E = \frac{.8 - .65}{2} = .075 \approx 7.5\%$

$\hat{p} = \frac{.8 + .65}{2} = .725 \approx 72.5\%$

Nov 15-12:31 PM

Confidence Interval for population mean μ:

$$\bar{x} - E < \mu < \bar{x} + E$$

Sample Mean

Point-estimate

Margin of error

Case I: σ Known

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

TI:

STAT TESTS Z Interval

inpt: Stats

Nov 15-12:51 PM

A Sample of 25 Students had a mean age of 30 Yrs. $n=25$, $\bar{x}=30$

C-level: .9

Find 90% Conf. interval for the mean age of all students assuming $\sigma = 8$ Yrs.

σ Known

$$\bar{x} - E < \mu < \bar{x} + E$$

$$30 - 2.6 < \mu < 30 + 2.6$$

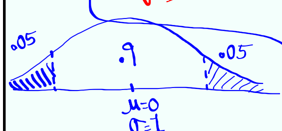
$$27.4 < \mu < 32.6$$

Since $\bar{x}$ whole #

$$\boxed{27 < \mu < 33}$$

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$= 1.645 \cdot \frac{8}{\sqrt{25}} = \boxed{2.6}$$



$Z_{\alpha/2} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

σ Known

Z Interval

inpt: Stats

$\sigma = 8$

$\bar{x} = 30$

$n = 25$

C-level: .9

Calculate

$$27 < \mu < 33$$

$$E = \frac{33 - 27}{2} = \boxed{3}$$

$$\bar{x} = \frac{33 + 27}{2} = \boxed{30}$$

Nov 15-12:56 PM

Standard deviation of Salaries of all nurses is Known to $400. $\sigma = 400$

A Sample of 30 nurses had a mean monthly Salary of $7500. $n = 30$, $\bar{x} = 7500$

NO C-level

Find Conf. interval for the mean Salary of all nurses. → Use .95

σ Known

Z Interval

inpt: Stats

σ: 400

whole # → $\bar{x}$: 7500

$n = 30$

C-level: .95

Calculate

$7357 < \mu < 7643$

$E = \frac{7643 - 7357}{2} = \boxed{143}$

$\bar{x} = \frac{7643 + 7357}{2} = \boxed{7500}$

Nov 15-1:06 PM

Confidence Interval for population mean μ:

$$\bar{x} - E < \mu < \bar{x} + E$$

Sample Mean / Point-estimate → $\bar{x}$

Margin of error → E

Case I: σ Known	Case II: σ unknown
$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ $df = n - 1$
TI: STAT TESTS ZInterval inpt: Stats	TI: STAT TESTS TInterval inpt: Stats

Nov 15-12:51 PM

Given: $n=15$, $\bar{x}=32.5$, $S=6.5$

C-level: .98

σ unknown

Find Conf. interval for pop. mean.

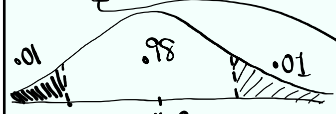
$E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$

$\bar{x} - E < \mu < \bar{x} + E$

$32.5 - 4.4 < \mu < 32.5 + 4.4$

$= 2.624 \cdot \frac{6.5}{\sqrt{15}} = \boxed{4.4}$

$\boxed{28.1 < \mu < 36.9}$



$\mu=0$

σ unknown

$df = n-1 = 14$

$t_{\alpha/2} = \text{invT}(.99, 14) = \boxed{2.624}$

σ unknown

T Interval

inpt: Stats

$\bar{x}=32.5$

$S=6.5$

$n=15$

C-level: .98

Calculate

Nov 15-1:16 PM

I randomly Selected 12 exams, here are the Scores.

75	82	68	90
95	100	70	88
80	65	100	58

Store in L1

use 1-Var Stats

$\bar{x}=80.9$

$S=14.0$

Round to 1-dec.

.95

No C-level

Find Conf. interval for the mean of all exams

σ Unknown → T Interval

inpt: Stats

Round to 1-dec. → $\bar{x}=80.9$

$S=14.0$

$n=12$

$df=11$

C-level: .95

Calculate

$72.0 < \mu < 89.8$

$E = \frac{89.8 - 72.0}{2} = \boxed{8.9}$

$\bar{x} = \frac{89.8 + 72.0}{2} = \boxed{80.9}$

Nov 15-1:25 PM

How to determine minimum $\underbrace{\text{Sample Size}}_{n}$:

Proportion

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$ with some algebra

$$n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

If decimal → Round -up

If $\hat{p}$ & $\hat{q}$ are both unknown, use .5 for each

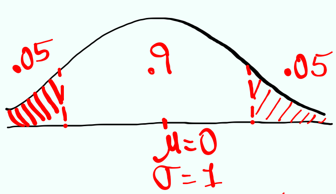
$$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

Nov 15-1:47 PM

Find minimum Sample Size needed to construct 90% Conf. interval for pop. prop. and margin of error not to exceed 6%

1) Assume $\hat{p} = .4$

$$n = \hat{p}\,\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$



$$= (.4)(.6)\left(\frac{1.645}{.06}\right)^2$$

$$= 180.401$$

$Z_{\alpha/2}$ = invNorm(.95,0,1) = $\boxed{1.645}$

$\boxed{n \approx 181}$

2) Assume $\hat{p}$ & $\hat{q}$ are both unknown

$$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{1.645}{.06}\right)^2 = 187.918$$

$\boxed{n \approx 188}$

Nov 15-1:51 PM

Find min. Sample Size needed to construct

NO C-level → .95

$\boxed{\text{Conf. interval}}$ for pop. proportion with margin of error not to exceed 8%.

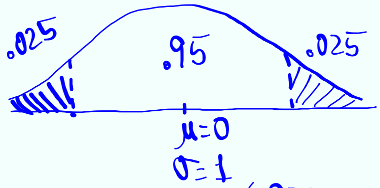
1) Assume $\hat{p} = .25$

$$n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

$$= (.25)(.75)\left(\frac{1.960}{.08}\right)^2$$

$$= 112.547$$

$$\boxed{n \approx 113}$$



$Z_{\alpha/2} = \text{invNorm}(.975, 0, 1) = \boxed{1.960}$

2) Assume $\hat{p}$ & $\hat{q}$ are both unknown

$$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{1.960}{.08}\right)^2 = 150.0625$$

$$\boxed{n = 151}$$

Nov 15-1:59 PM

How to determine minimum $\underbrace{\text{Sample Size}}_{n}$:

Population Mean

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

with some algebra

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

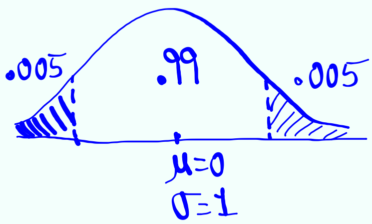
If decimal → Round-up

If σ is unknown, use S instead.

$$n = \left(\frac{Z_{\alpha/2} \cdot S}{E}\right)^2$$

Nov 15-1:47 PM

Find min. Sample Size needed to construct 99% Conf. interval for pop. mean with $\sigma = 25$ and $E = 10$.



$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

$z_{\alpha/2}$ = invNorm(.995,0,1) $= \boxed{2.576}$

$$= \left(\frac{2.576 \cdot 25}{10}\right)^2$$

$= 41.474$

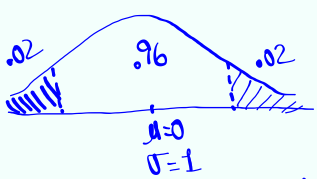
$\boxed{n \approx 42}$

Nov 15-2:10 PM

Find min. Sample Size needed to construct 96% Conf. interval for pop. mean and error not to exceed 8 and $S = 12.5$.

Missing σ, use S

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$



$$= \left(\frac{2.054 \cdot 12.5}{8}\right)^2$$

$z_{\alpha/2}$ = inv Norm(.98, 0, 1) $= \boxed{2.054}$

$= 10.3\ldots$

$\boxed{n \approx 11}$

Redo with $E = 5$

$n = \left(\frac{2.054 \cdot 12.5}{5}\right)^2 = 26.368$ $\boxed{n \approx 27}$

Nov 15-2:14 PM

Given: $n = 15$ $\quad \bar{x} = 28$ $\quad \boxed{s = 8}$

1) Find $\boxed{\text{Conf. interval}}$ for pop. mean.

NO C-level $\Rightarrow .95$

σ unknown

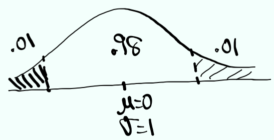
T Interval

$$\boxed{24 < \mu < 32}$$

2) Find margin of error

$$E = \frac{32 - 24}{2} = \boxed{4}$$

3) Find min. Sample Size needed to construct 98% Conf. interval for pop. mean and error not to exceed 2.5.



$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

$z_{\alpha/2} = \text{invNorm}(.99, 0, 1)$

$$= \left(\frac{2.326 \cdot 8}{2.5} \right)^2$$

$$\boxed{n \approx 56}$$

Redo with $E = 10$

$$n = \left(\frac{2.326 \cdot 8}{10} \right)^2 \Rightarrow \boxed{n = 4}$$

SG 22 & 23 ✓

Nov 15-2:22 PM